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Analysis of a phononic crystal constituted of piezoelectric layers using electrical impedance measurement

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Abstract

Active materials are investigated as a tuning possibility for phononic crystals. Electromechanical properties of piezoelectric materials electrical and mechanical variables are coupled. Thus, the propagation properties of elastic waves can be tuned using electrical impedance load connected to the piezoelectric layer. In this study, a theoretical one-dimensional model is proposed to calculate the electrical impedance of an active layer located inside a finite periodic structure including both piezoelectric and passive layers. Depending on the electrical impedance load, various effects on the position and the amplitude of the electrical resonance are observed in very good agreement, between experimental and theoretical results.

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Keywords: Phononic crystal, piezoelectric material, electrical impedance, ultrasonic waves.

1. Introduction

Phononic crystals (PC) present special guiding and filtering properties. The range of applications of PC is widened with the use of active materials, enabling the control of their properties. The electrical impedance of a piezoelectric layer is related to the characteristics of the surrounding layers. This study deals with the electrical impedance tuning and measurement of passive/active periodic structures, including electrically tuned piezoelectric layer. A thickness mode model is developed to determine the electrical impedance of a piezoelectric layer integrated inside a finite multilayer structure. The piezoelectric thickness mode resonance can be modified by an external electrical load as it has been demonstrated by Prokic [1]. In this study, the effect of an electrical load applied on a piezoelectric layer constituting a PC is investigated as a tuning possibility.

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2. Constitutive equations

In this section, the theoretical basis of this study is developed. The fundamental equation of piezoelectricity is expressed in the case of a pure thickness mode. This assumption is only valid since the lateral dimensions are greater than the thickness as reminded by Lethiecq et al. [2]. Moreover, the electrodes on both faces of the piezoelectric layer are considered thin enough to be neglected in the mechanical point of view. As a result, a piezoelectric layer is reduced to one-dimensional electromechanical material:

$$\begin{cases} T_3(z) = c_{33}^D S_3(z) - h_{33} D_3(z) \\ E_3(z) = -h_{33} S_3(z) + \beta_{33}^S D_3(z) \end{cases} \tag{1}$$

where (T_3, S_3) are the mechanical stress and strain, respectively, (E_3, D_3) are the electrical field and displacement, respectively. The piezoelectric properties are: $c_{33}{}^D$ the elastic constant at constant displacement, $\beta_{33}{}^S$ the dielectric permeability constant at constant strain and h_{33} the piezoelectric constant. In this study, only longitudinal waves are considered along the z direction. When the electrical conditions are considered inside the material itself, according to Gauss's law applied on the dielectric element, the electrical displacement $D_3(z)$ is considered independent of the position z. Nevertheless, when an additional outer electrical impedance Z_a is connected in parallel to the piezoelectric plate (Fig. 1 (a)), the electrical displacement D_3 can be directly expressed as a function of the outer electrical impedance Z_a [3-4]:

$$D_3(Z_a) = \frac{h_{33}}{\beta_{33}^S h.\left(1 + \dfrac{Z_a}{Z_0}\right)} \left(u_3(h) - u_3(0)\right) \tag{2}$$

where $u_3(z)$ is the mechanical displacement along the z direction, and $Z_0 = 1/(j\omega C_0)$ is the electrical impedance corresponding to the capacitive effect $C_0 = A_p/(\beta_{33}{}^S h)$.

Firstly considered electrically in open-circuit, the constitutive parameter dependency on the position z (eq. 1) is to be revised due to the outer electrical impedance Z_a, which causes a new dependency (eq. (2)):

$$\begin{cases} T_3(z, Z_a) = c_{33}^D S_3(z) - h_{33} D_3(Z_a) \\ E_3(z, Z_a) = -h_{33} S_3(z) + \beta_{33}^S D_3(Z_a) \end{cases} \tag{3}$$

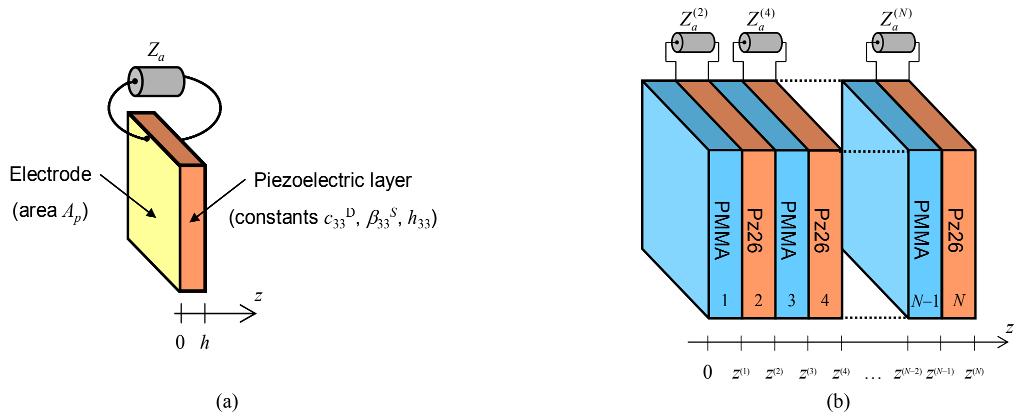

Fig. 1: (a) Elementary piezoelectric layer with electrodes on both faces, loaded by an outer electrical impedance Z_a; (b) Periodic structure composed of an elementary stack made of a passive layer and a piezoelectric one.

3. Multilayer structure

3.1. General case

Here, a multilayer periodic structure composed of n_p periods of a bi-layer elementary stack ($n_s = 2$) made of a passive and piezoelectric layers is studied (Fig. 1 (b)). In order to obtain a noticeable acoustic impedance contrast (Table 1), we chose an elementary stack made of (PMMA/Pz26). Each constitutive layer is being numbered from 1 to N, where N is the total number of layers resulting from the product of the number of stacks n_s, by the total number of periods $n_p = N/n_s$.

Table 1. Acoustical properties of the constituting layers.

Material	h (mm)	ρ (kg/m^3)	c_L (m/s)	Z (MRa)
PMMA	4.13	1140	2740	3.12
Pz26	1.00	7700	4530	34.9

h: thickness; ρ: density; c_L: longitudinal velocity; Z: acoustical impedance.

In this multilayer configuration, all the properties mentioned before (eq. (1) and (2)) are now particularized to each constitutive layer n, by an exponent (n). The displacement in the layer numbered n is given by:

$$u_3^{(n)}(z) = A^{(n)}.e^{+jk^{(n)}z} + B^{(n)}.e^{-jk^{(n)}z} \tag{4}$$

where $A^{(n)}$ and $B^{(n)}$ are the standing waves amplitude coefficients of the forward and backward components, $k^{(n)} = \omega / v^{(n)}$ is the wavenumber associated to the layer numbered n.

The mechanical boundary and continuity equations at each interface results in:

$$\begin{cases} T_3^{(1)}(0) = 0 \\ T_3^{(N)}(h_N) = 0 \end{cases} \quad \text{and} \quad \begin{cases} T_3^{(n)}(h_n) = T_3^{(n+1)}(0) \\ u_3^{(n)}(h_n) = u_3^{(n+1)}(0) \end{cases} \quad \text{for} \quad n \in [1, N-1] \tag{5}$$

Thus, a global matrix [M] is expanded as a function of the [AB] vector made of the $A^{(n)}$ and $B^{(n)}$ coefficients (eq. (4)) and the [SM] vector constituted of zeroes, excepted the indexes ($2m$) and ($2m$+1) which values are equal to one:

$$[M].[AB] = D_3^{(m)} h_{33}^{(m)}.[SM] \quad \text{and} \quad \begin{cases} c_A^{(m)} = A^{(m)} / D_3^{(m)} \\ c_B^{(m)} = B^{(m)} / D_3^{(m)} \end{cases} \tag{6}$$

where m is the piezoelectric layer whose electrical impedance is measured and for which $Z_a^{(m)}$ is considered infinite. As a result, complex and frequency dependent variables $c_A^{(m)}$ and $c_B^{(m)}$ are deduced from the properties and boundary conditions of the constituting layers. After some rearrangements, we obtain a formula similar to that of passive surrounding layers (Maréchal (2007)), involving the input impedances of the rear and front faces:

$$Z_e^{(m)} = Z_0^{(m)}.\left(1 - \left(k_t^{(m)}\right)^2 \frac{c_{33}^{D,(m)}}{h^{(m)} h_{33}^{(m)}} \left(c_A^{(m)}\left(e^{+jk^{(m)}h^{(m)}} - 1\right) + c_B^{(m)}\left(e^{-jk^{(m)}h^{(m)}} - 1\right)\right)\right) \tag{7}$$

3.2. Case of study

A finite number of periods were assembled in view to validate the general analysis developed in the previous subsection. The chosen configuration is constituted of two periods, i.e. four layers, the second and the fourth being piezoelectric. The electrical impedance (eq. (7)) was measured on the second one ($m = 2$) and the electrical load $Z_a^{(4)}$ was varied on the fourth one (Fig. 2 (a)). The corresponding dispersion curve is shown in Fig. 2 (b).

These dispersion curves (Fig. 2 (c)) are compared in good agreement with the transmission coefficient through a ten layers structure (Fig. 2 (d)), as illustrated by the correspondence of the stop bands.

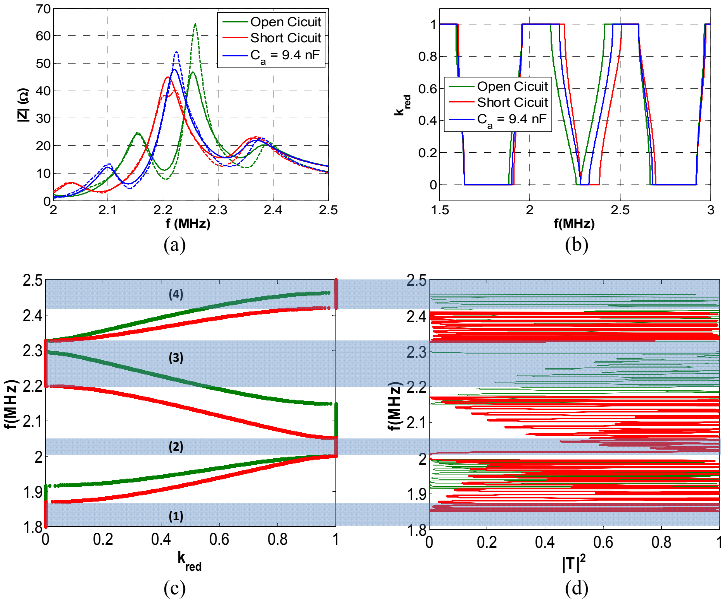

Fig. 2: (a) Electrical impedance modulus $|Z_e^{(2)}(f)|$ of the four layer structure; (b) dispersion curves of the resulting PC with $Z_a^{(4)} \to \propto$, $Z_a^{(4)} \to 0$, $Z_a^{(4)} = 1/(j\omega C_a)$; (c) dispersion curves and associated (d) transmission coefficients $|T|^2$ of a ten layers structure with $Z_a^{(4)} \to \propto$, $Z_a^{(4)} \to 0$.

4. Conclusion

In this study, the constitutive equations of the piezoelectricity where used to propose an analytical model in order to calculate the electrical impedance of an active layer within a phononic crystal. This general model takes into account each electrical outer impedance connected in parallel to each piezoelectric layer, excepting that for which the impedance is evaluated. The measured electrical impedance across a range of frequencies shows some significant changes depending on the considered electrical impedance load. These changes can be related to modifications in the band structure of the acoustic transmission through the PC. As a perspective, the proposed formalism in this study can be used as matching structure for the ultrasonic transducer development.

Acknowledgements

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