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***Abstract*—Model Predictive Control (MPC) is based on the concept of receding horizon, the future output prediction, and the minimization of a cost function to provide the optimal control sequence. MPC controller contains three parameters: a control horizon N_c, a prediction horizon N_p and a weighting factor λ. A successeful implementation of MPC requires an appropriate setting of these parameters. In this paper, an analytical approach for tuning the control horizon is presented while taking into account constraints. The idea of our novel approach consists on computing the value of the optimal control horizon in such a way it ensures the numerical stability. The interest of our approach is to be applicable to a wide set of linear controllable Single-Input Single-Output (SISO) processes whatever their orders. The issues of numerical condition and closed-loop stability are addressed in this paper. The proposed approach is tested via a simulated pH neutralization process. Results are compared to emphasize the effectiveness of the proposed approach.**

***Keywords*—Predictive control, control horizon tuning, analytical approach, effective rank, condition number, stability guarantee, SISO system.**

I. Introduction

MPC has proven to be an excellent candidate for controlling complex systems and is now widely implemented in industry for many years [1], [2]. Its parameters N_c, N_p and λ influence significantly the closed-loop behavior, stability and robustness in a complex manner [3], [4].
MPC parameters tuning is, therefore, a challenging issue. Over the last decades, many research efforts have led to the development of MPC tuning approaches, [5], [6], [7]. We can classify these methods on three categories:

- The first category includes the analytical methods (more or less numerical-based). Only few work has been published due to the complexity of the problem [8], [9], [10], [11], [12].
- The second category includes the heuristic methods. It aims to find an approximation of optimal values for the MPC. Different approaches have been extensively published in the literature. Some of them are based on fuzzy logic [13], [14], [15] on genetic algorithms [16], [17] and on neural networks [18], [19]. Other choose to use the Analysis of Variance (ANOVA) and nonlinear regression [20].
- The third category includes the empirical approaches where the MPC parameters are determined regarding the designer's experience [21], [22], [23], [24].

The majority of these methods have a trial-and-error nature, which does not always permit to identify explicitly the robustness area of the control system. None of the published methods deals with a general case of a process whatever its order and none of these methods offers a setting of the control horizon while taking into account the constraints. In this paper, we intend to overcome these limits thanks to our original approach by computing the control horizon while enhancing the numerical condition of the controlled process regarding an analytical method. Important abilities of our approach are highlighted when linear inequality constraints have to be considered. Indeed, the tuning strategy easily integrates the constraints without increasing the calculation effort or undoing the proposed optimization algorithm. Its advantage lies on the fact that it is applicable to any controllable and observable SISO linear system.

This paper is outlined as follows: section II reminds the theoretical background on MPC based on state-space representation. Section III highlights the proposed analytical tuning approach for the control horizon. In section IV, a comparative study of the obtained performances on a simulated pH neutralization process is carried out. Simulations results are shown to emphasize the effectiveness of our approach.

II. Remind on Model Predictive Control Design

A. Augmented state-space model

Let consider a SISO system represented by the following discrete-time state-space model:

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m u(k) \\ y(k) &= C_m x_m(k) \end{aligned} \tag{1}$$

Where $y \in \mathbb{R}$ is the output system, $u \in \mathbb{R}$ the manipulated variable and the row matrix x_m is the state variable vector of size n_{A_m}. k is the sampling instant (positive value). In (1), $A_m \in \Re^{n_{A_m} \times n_{A_m}}$, $B_m \in \Re^{n_{A_m} \times 1}$ and $C_m \in \Re^{1 \times n_{A_m}}$ are the state-space matrices.
In order to design predictive controller, let adopt the formulation of augmentated-state-model with embedded integrators whose advantages have been already discussed [25]. Such a formulation has been inspired by the integral functionality [5]. The difference of the state and control variables are:

$$\begin{aligned} \Delta x_m(k) &= x_m(k) - x_m(k-1) \\ \Delta u(k) &= u(k) - u(k-1) \end{aligned} \tag{2}$$

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By integrating the increments of the variables $x_m(k)$ et $u(k)$. Equation (2) becomes:

$$\begin{aligned}\Delta x_m(k+1) &= A_m(x_m(k) - x_m(k-1)) \\ &+ B_m(u(k) - u(k-1)) \\ &= A_m \Delta x_m(k) + B_m \Delta u(k).\end{aligned}$$

Now, let consider a new state space vector as:

$$x(k) = \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}$$

Finally, the augmented state-space model is as following:

$$\overbrace{\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix}}^{x(k+1)} = \overbrace{\begin{bmatrix} A_m & 0_m^t \\ C_m A_m & 1 \end{bmatrix}}^{A} x(k) + \overbrace{\begin{bmatrix} B_m \\ C_m B_m \end{bmatrix}}^{B} \Delta u(k)$$

$$y(k) = \overbrace{\begin{bmatrix} 0_m & 1 \end{bmatrix}}^{C} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} \quad (3)$$

Where $0_m = [0 \quad 0 \cdots \quad 0]$ is a row matrix of size n_{A_m}. The matrices A, B and C of size respectively $(n_A \times n_A)$, $(n_A \times 1)$ and $(1 \times n_A)$ (with $n_A = n_{A_m} + 1$), constitute the discrete time augmented state-space representation.

B. MPC formulation

As an hypothesis, the system is supposed to be observable and controllable. The incremental control signal vector ΔU of size $(1 \times N_c)$ is defined as:

$$\Delta U = \begin{bmatrix} \Delta u(k) & \Delta u(k+1) & \cdots & \Delta u(k+N_c-1) \end{bmatrix}^t \quad (4)$$

While superscript t denotes matrix transpose.
The desired output Y_{des} of size $(N_p \times 1)$ is:

$$Y_{des} = \begin{bmatrix} y_{des}(k+1) & y_{des}(k+2) & \cdots & y_{des}(k+N_p) \end{bmatrix}^t$$

Assuming the predicted output vector $\hat{Y}$ defined by [25]

$$\hat{Y} = Fx(k_i) + \Phi \Delta U \quad (5)$$

where

$$\hat{Y} = \begin{bmatrix} \hat{y}(k+1 \mid k) & \hat{y}(k+2 \mid k) & \cdots & \hat{y}(k+N_p \mid k) \end{bmatrix}^t,$$

$$F = \begin{bmatrix} CA & CA^2 & CA^3 & \cdots & CA^{N_p} \end{bmatrix}^t, \quad (6)$$

and

$$\Phi = \begin{bmatrix} CB & 0 & 0 & ... & 0 \\ CAB & CB & 0 & ... & 0 \\ CA^2B & CAB & CB & ... & 0 \\ \vdots & & & & \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & ... & CA^{N_p-N_c}B \end{bmatrix} \quad (7)$$

Let the cost function J to be minimized as follows:

$$J = (Y_{des} - \hat{Y})^t (Y_{des} - \hat{Y}) + \Delta U^t \bar{R} \Delta U \quad (8)$$

where $\bar{R}$ is a $(N_c \times N_c)$ matrix defined by:

$$\bar{R} = \lambda I_{(N_c \times N_c)} \quad (9)$$

and I denotes the identity matrix.
In order to calculate the optimum value of the manipulated variable, let the partial derivation of J with respect to ΔU as:

$$\frac{\partial J}{\partial \Delta U} = -2\Phi^t(Y_{des} - Fx(k)) + 2(\Phi^t \Phi + \bar{R})\Delta U \quad (10)$$

The optimal control sequence ΔU is found by solving (10) equals to zero, which leads to:

$$\Delta U = (\Phi^t \Phi + \bar{R})^{-1} \Phi^t (Y_{des} - Fx(k)) \quad (11)$$

According to the receding horizon principle, the control signal applied to the process will be the first element of ΔU as:

$$\begin{aligned} u(k) &= u(k-1) + \Delta u(k) \\ u(k) &= u(k-1) + I_{(1 \times N_c)} (\Phi^t \Phi + \bar{R})^{-1} \Phi^t (Y_{des} - Fx(k)) \end{aligned}$$

Note that the estimation of the optimal control requires the state vector $x(k)$. Here the control signal is subject to linear inequality constraints as proposed by [25]:

$$U^{min} \leq U \leq U^{max}. \quad (12)$$

III. Analytical Tuning of the control Horizon

This section is devoted to explain the analytical method we propose to tune N_c.

Most often in the literature, N_c is taken equal to unity as proposed in the guidelines provided by [5] and lower than the predicted horizon N_p. In fact, considering $N_c = 1$ gives acceptable control performance. A higher N_c is recommended when systems to control have unstable poles [5].

A. The Hessian condition number

In order to compute analytically the optimal value of N_c, the concept of numerical stability is considered. Indeed, the numerical stability concerns mainly the condition number of a square nonsingular matrix [26]. Generally speaking, this specific matrix is the Hessian matrix (or just the Hessian) of an algorithm. Dealing with MPC, the Hessian matrix is present in the formulation of the optimal control sequence [27]. With the assumption that $(\Phi^t \Phi + \bar{R})^{-1}$ exists, let consider the Hessian matrix H of size $(Nc \times Nc)$ defined as:

$$H = (\Phi^t \Phi + \bar{R})^{-1} \quad (13)$$

To evaluate the conditioning of the Hessian matrix H, one calculates its condition number defined by:

$$\begin{aligned} cond(H(k)) &= \|H(k)\|_2 \cdot \left\|H(k)^{-1}\right\|_2 \\ &= \frac{\sigma_{max}(k)}{\sigma_{min}(k)} \end{aligned} \quad (14)$$

Where $\sigma_{max}(k)$ and $\sigma_{min}(k)$ are respectively the maximum and the minimum singular values of matrix H. Then, the condition number of a matrix indicates how close a matrix is to be singular: a matrix with a large condition number is nearly singular, wheras a matrix with a condition number close to unity is far from being singular [26].

Regarding the literature, different ways exist to improve the condition number of a Hessian matrix: thus [28] suggests using a numerical stable projection or re-writing the Hessian formulas. [29] improves the condition number of the Hessian matrix using a latent variable method with MPC. [30] establishes a strategy combining a singular value decomposition (SVD) and a receding horizon control (RHC) principle to enhance the Hessian conditioning. A sub-optimal control signal is produced by discarding the smallest singular values of the Hessian. De Keyser recommends solving this numerical problem using principal component analysis (PCA) [27]. This method looks like the SVD-RHC technique of [30]. Both approaches cannot systematically leads to satisfactory results in comparison with the method proposed in [25], [31] and [32]. This one uses an exponential data weighting in the cost function to enhance the Hessian conditioning. This additive weighting technique yields a more straightforward and practical method for engineers and researchers. Here in this paper, we intend to overcome these limits and enhance the condition number of the Hessian matrix by computing analytically the control prediction.

B. Improving the condition number: a dimension reduction problem

As shown in [33] and [34], high MPC horizons guarantee the system closed-loop stability. Ideally, these values tend towards infinity ($N_c \mapsto \infty, N_p \mapsto \infty$). Then, we notice that the issue of improving the condition number of the Hessian matrix is converted into a dimension reduction problem of matrix H. Thus, when the dimension reduction problem is solved, an optimal value of N_c is deduced (Figure 1).
In order to reduce the dimension of matrix H, the concept of the effective rank (ER) is considered [35] (Appendix 1).

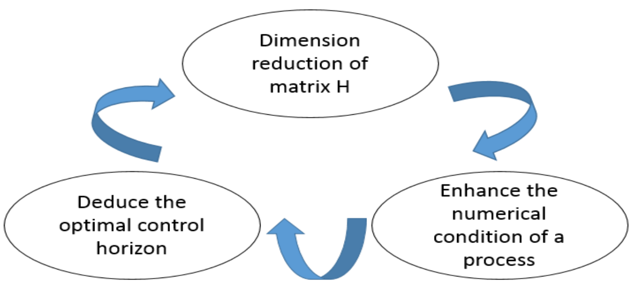

Fig. 1. Proposed N_c tuning strategy

C. Relation between the Effective Rank and N_c

In this part, we relate the concept of the effective rank with MPC design in order to evaluate the optimum value of the control horizon. The different steps for calculating the optimal value of N_c are as follows:

1) Tending $N_c \mapsto \infty$ and $N_p \mapsto \infty$ ($Nc < Np$, $N_c \in \mathbb{N}^*$ and $N_p \in \mathbb{N}^*$).
2) Taking $A_{ER} = H$.
3) Evaluating Q defined as follows:
$$Q = min\{M_{ER}, N_{ER}\} = min\{Nc, Nc\} = Nc.$$
4) Decomposing into singular values the matrix H and evaluating the matrix $\sigma = [\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_\infty]^t$:
$$H = U_H D_H V_H \tag{15}$$
$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_\infty \tag{16}$$
5) Evaluating the singular value distribution p_k with $k = [1, \quad 2, \quad \cdots, \quad \infty]$.
6) Computing the Shannon entropy defined in (27).
7) Evaluating finally the optimal value of N_c by solving the following minimization problem using Yalmip toolbox [36]:
$$\begin{cases} N_{c_{opt}} = round(e^{H_{Shannon}(p_1, p_2, \cdots, p_\infty)}) \\ \quad min \begin{cases} cond(H(k)) - 1 \\ N_{c_{opt}} \end{cases} \end{cases} \tag{17}$$
Under the following constraints:
$$\begin{cases} N_c \in \mathbb{N}^* \\ 1 \leq N_c < N_p \\ U^{min} \leq U \leqslant U^{max} \end{cases}$$

As a practical advantage, this strategy does not require an apriority knowledge of the minimum rank and it takes no account of the full singular value spectrum [35]. As a conclusion, the smallest control prediction we take, the better condition number is. Thus, the numerical stability is enhanced.
Note 1 Linear inequality constraints considered in the MPC do not influence the tuning approach proposed in this paper.
Note 2 [25] shows that the minimization of MPC horizons enhance the numerical condition of the controlled process. One way to reduce the control horizon is using the concept of the effective rank.

IV. Application to a simulated chemical process

As a benchmark, a simulated pH neutralization process is considered in this part [37], [38].

A. Performances criteria

In order to carry out a comparative study between the conventional guideline proposed by Shridhar [8] and our approach, the following performance indexes are considered:

1) **The stability degree index (SDI)** is used to evaluate the system closed-loop stability. It is the difference between the radius of the unit circle and the modulus of the pole most remote from the unit circle centre [14]:
$$SDI(k) = 1 - max(|p_1|, |p_2|, \cdots, |p_{A_N}|). \tag{18}$$
where $\{p_1, p_2, \cdots, p_{A_N}\}$ are the eigenvalues of $A_{cl} = A - BK_{mpc}$. So the closed-loop system is stable when $SDI \in]0, 1[$.
2) **The variance of control signal (VARU)** makes it possible to observe the mean value of the square deviations of u(k) from its average as follows [39]:
$$VARU(k) = \overline{u(k)^2} - \left[\overline{u(k)}\right]^2 \tag{19}$$
where $\bar{u}$ is the mean of u.
3) **The rise time (RT)** is from 10% to 90% [39].
4) **The settling time (ST)** is within 2%.
5) **The overshoot (OV)** is the overshoot in the output signal.
6) **The static error (SE)**
7) **The control signal energy (CSE)** is:
$$CSE = \sum_{k=0}^{ST} u^2(k) \tag{20}$$

8) **The control effort energy (CEE)** is:

$$CEE = \sum_{k=0}^{ST} \Delta u^2(k) \tag{21}$$

9) **The computational load (CL)** depends heavily on the control horizon [39].
10) **the condition number of H (Cond(H)** is computed from equation (14).

B. Process description

The considered benchmark is a nonlinear multivariable system. For simplification reasons, we deal with a SISO pH neutralization process. The process consists of acid, base and buffer streams mixed in a vessel. A schematic diagram of the studied process is presented in Figure 2.

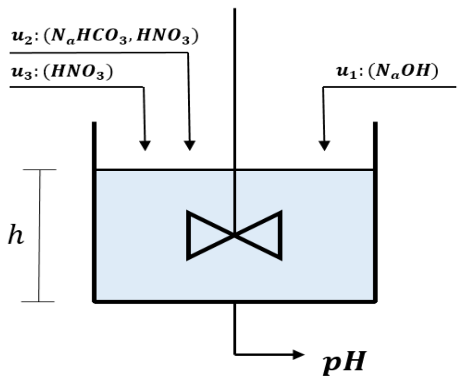

Fig. 2. Sketch of the pH neutralization process

In the SISO case, acid (HNO_3) stream represents the measured system disturbance, the base ($NaOH$) stream is the control signal and the buffer ($NaHCO_3, NaOH$) stream represents the unmeasured disturbance of the system.

The main objective of this part is to control the value of the pH of the outlet stream. In addition, it is assumed that the pH of the outlet stream is measured at a distance from the plant, which introduces a measurement time delay θ. The sampling time T_s is set to $8\ ms$ as in [40].

C. Process linearisation

Since the SISO system is highly nonlinear and for control purpose, we adopt the linear model constructed by [40] around an operation point ($pH = 7$) whose the augmented-state representation in the discrete time domain is the following:

$$A = \begin{bmatrix} 0.9102 & 0 \\ 0.0397 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0.0436 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}. \tag{22}$$

D. Tuning parameters

The prediction horizon and the weighting factor values are chosen according to [8]. Based on the tuning approach described in Section III, the control horizon is analytically computed as follows:

- Initializing $N_c = 39$, $N_p = 40$ and $\lambda = 0.063$.

TABLE I. MPC TUNING GUIDELINES

	Shridhar and Cooper 1997 [8]	Proposed approach
$T_s(s)$	$T_s \le \frac{\tau}{10}$ *and* $\frac{\theta}{2}$	$T_s \le \frac{\tau}{10}$ *and* $\frac{\theta}{2}$
N_c	Integer, from 1 to 6	$e^{H_{Shannon}(p_1,p_2,\cdots,p_\infty)}$
N_p	$\frac{5\tau}{T_s} + \frac{\theta}{T_s} + 1$	$\frac{5\tau}{T_s} + \frac{\theta}{T_s} + 1$
λ	fK^2	fK^2
f	$\begin{cases} 0 & N_c = 1 \\ \frac{N_c}{500}\left(\frac{3.5\tau}{T_s} - \frac{N_c-5}{2}\right) & N_c > 1 \end{cases}$	$\begin{cases} 0 & N_c = 1 \\ \frac{N_c}{500}\left(\frac{3.5\tau}{T_s} - \frac{N_c-5}{2}\right) & N_c > 1 \end{cases}$

- Decomposing H into singular values, evaluating σ and computing the Shannon entropy value based on (15), (16) and (27) as:

$$H_{Shannon} = 0.5530$$

- Computing the optimal value of N_c based on (17) as:

$$\begin{aligned} N_{c_{opt}} &= e^{H_{Shannon}(p_1,p_2,\cdots,p_\infty)} \\ &= 1.7385 \simeq 2 \end{aligned}$$

E. Simulation test

The tuning strategy detailed in [8] propose guideline for tuning a First Order Plus Dead Time (FOPDT) model whose transfer function is, in the general case as follows:

$$G_{FOPDT}(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \tag{23}$$

The proposed tuning equations are shown in Table I. The simulation results are depicted in Figure 3. The parameters

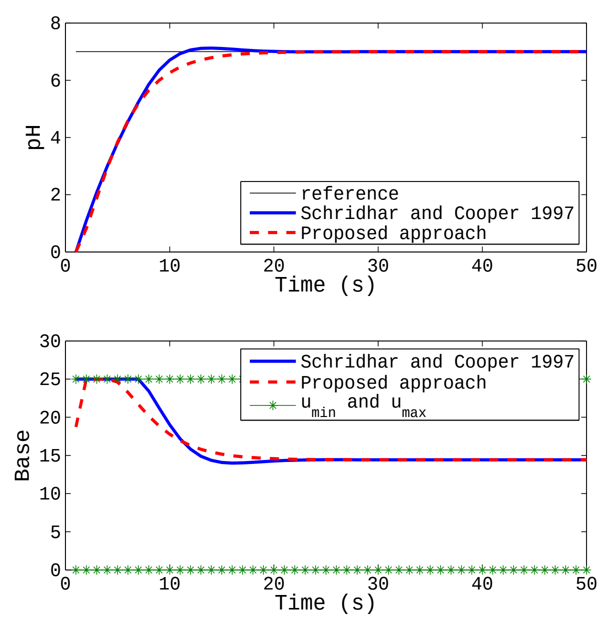

Fig. 3. Output and control signals vs. time

used in simulations and performance comparisons can be made from Table II.

The linear inequality constraint of the control signal is taken as:

$$0\ ml/s \leq u \leq 25\ ml/s. \tag{24}$$

TABLE II. MPC PARAMETERS AND PERFORMANCE COMPARISONS OF MPC TUNING METHODS FOR THE PH NEUTRALIZATION PROCESS

	Shridhar and Cooper 1997 [8]	**Proposed approach**
$T_s(ms)$	8	8
N_c	4	2
N_p	40	40
λ	0.063	0.063
SDI	0.2850	0.2789
VARU	37.3001	12.7142
RT (s)	7.2537	8.3255
ST (s)	14.2457	15.2734
OV (%)	1.7938	0
SE	0.6159	0.2965
CSE $(e+03)$	6.3796	4.8804
CEE $(e+03)$	4.3775	0.4237
CL	5.8155	4.6622
Cond(H)	352.3533	1.7244

As shown in Figure 3, the fastest response is obtained using Schridhar approach [8]. However, this latter leads to the highest VARU, CSE and CEE indices. The best SDI is obtained thanks to the guideline of [8] but this method causes a high overshoot and a considerable static-error. The condition number computed with our approach is about 200 times smaller than the one given by [8] which indicate a remarkable improvement in the calculation conditions. In conclusion, only with the computation of the control horizon, few performance criteria have been greatly improved. In the work perspective, an analytical method for calculating the prediction horizon and the weighting factor is expected.

V. CONCLUSION

An analytical approach with an enhancement of numerical condition to tune the control horizon value has been proposed in this paper. This approach is dedicated to nonlinear systems around the operating points. Some advantages of our novel approach are highlighted when constraints have to be considered. Indeed, the tuning strategy easily integrates the linear inequality constraints without increasing the computational effort or undoing the proposed optimization algorithm. Future work will aim to find an analytical approach for computing the prediction horizon and the weighting factor.

VI. APPENDIX 1

Effective Rank concept Let consider a complex-valued non-all-zero matrix A_{ER} of size $(M_{ER} \times N_{ER})$ whose SVD is given by $A_{ER} = U_{ER} D_{ER} V_{ER}$. Where U_{ER} and V_{ER} are unitary matrices of size $(M_{ER} \times M_{ER})$ and $(N_{ER} \times N_{ER})$, respectively, and D_{ER} is an $(M_{ER} \times N_{ER})$ diagonal matrix containing the real positive singular values:

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_Q \geq 0,$$

with $Q = min\{M_{ER}, N_{ER}\}$.
Let define for writing simplification $\sigma = \begin{bmatrix}\sigma_1 & \sigma_2 & \cdots & \sigma_Q\end{bmatrix}^t$ and the singular value distribution:

$$p_k = \frac{\sigma_k}{\|\sigma\|_1} \tag{25}$$

where $\|.\|_1$ denotes the l_1 norm defined as $\|\sigma\|_1 = \sum_{k=1}^{Q} |\sigma_k|$

Definition VI.1. Effective Rank
The effective rank of the matrix A_{ER} is defined as:

$$erank(A_{ER}) = \exp H_{Shannon}(p_1, p_2, \cdots, p_\infty) \tag{26}$$

where $H_{Shannon}(p_1, p_2, \cdots, p_Q)$ is the Shannon entropy:

$$H_{Shannon}(p_1, p_2, \cdots, p_Q) = -\sum_{k=1}^{Q} p_k \log p_k. \tag{27}$$

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