Development of the xTAN method for Cyber Physical Systems (CPS) under electromagnetic environment

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Summary

• Principles
• Illustration on a simple example in automatic
• To include human factor:
  – Game theory
  – Coupling TAN & game theory: xTAN method
• Very simple example: pedestrian versus smart vehicle problem
• Insert electromagnetic influence
• Prospective & Conclusion
Each edge is identical mathematically to a "primitive" object.

For example:
To make a system, edges are connected through meshes and meshes are coupled between them in a second step.

<table>
<thead>
<tr>
<th>Edges\meshes</th>
<th>J1</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>1</td>
</tr>
<tr>
<td>I2</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ Z_{\text{mesh}} = A(.) + B(.) \]

Tensorial analysis of networks: TAN
Gabriel Kron 1939
Current :: flux :: vector – natural space \( \vec{i} = i^k \vec{u}_k \)

Charge displacement

Work & potential difference

\[ v = \alpha \int_0^x f(i) \, dx \]

\[ \nu = \nu_k n^{*k} \]

\[ \delta_k^m = \vec{u}_k \cdot n^{*m} \]
Fundamental tensor

How to go from the natural space to the dual one?
Thanks to the fundamental tensor

\[ x_\alpha = Z_{\alpha \beta} x_\beta \]
\[ S = x^\alpha Z_{\alpha \beta} x_\beta \]

In the electrical case and for the mesh space: primal space = currents, dual space = electromotive force (voltages have disappeared)
Reported electromotive force depending on other current:

\[ U(\text{edge } k) = z(i \text{ edge } m) \]

\[ U(\text{mesh } k) = z(i \text{ mesh } m) \]
General approach

Fundamental principles of tensors is to keep relations under change of reference axes and to define projection into local tangent plan

\[ e_a = z_{ab} i^b \Rightarrow z_{ab} \Lambda^b_\sigma J^\sigma \]

\[ \Lambda^a_\beta e_a = \Lambda^a_\beta z_{ab} \Lambda^b_\sigma J^\sigma = z_{\beta\sigma} J^\sigma = e_\beta \]

Local projection
Changing base

Parameters = Flux: curvilinear coordinates

 Flux:
curvilinear coordinates

\[ e_a(J^b, t, ...) = z_{ab}(J^b, t, ...) \]
\[ \iff \tilde{v}^b = \frac{\partial e}{\partial i^b} \iff \overrightarrow{\omega}_b \]

\[ \begin{cases} g_{ab} = \langle \overrightarrow{\omega}_a | \overrightarrow{\omega}_b \rangle \\ e_a - \mathcal{L}_{ab} \partial_t J^b = g_{ab} J^b \end{cases} \]
Classical automatic block diagrams translated into xTAN graph

\[ y = G(x - Fy) \]

No distinction between the signals. Functions are defined in the blocks.

\[ y = \frac{G(x - Fy)}{1} \]

There are flux and efforts. A metric give the relation between them. Functions are defined in cords.
Classical electrical machine block diagrams

\[ g = \begin{bmatrix} R + Lp & -K_e \\ -K_t & K_d + Jp + \alpha \end{bmatrix} \]

\[
\begin{align*}
  u &= (R + Lp) i - K_e \omega \\
  -C_r &= -K_t i + (K_d + Jp) \omega
\end{align*}
\]

\[
\begin{align*}
  u_a &= g_{ab} f^b \\
  u_a &\in [v_m \ldots v_x]
\end{align*}
\]
The graph and its associated equations can change with time

Initial state

One evolution

Another evolution
To include human factor

\[ u_a = g_{ab} f^b \]
\[ u_a \in [v_m \ldots v_x] \]

\[ \tilde{u} = (C, T, G) \]

\[ \tilde{u}_1 = \hat{t}_{10} \tilde{u}_0 \]

\[ (C, T, G)_0 \]

\[ (C, T, G)_1 \]

\[ (C, T, G)_2 \]

Kuhn’s tree form

Payoff matrix form
Pedestrian versus smart vehicle problem

Example

\[
z = \begin{bmatrix} a & -f & 0 \\ C & b & 0 \\ 0 & 0 & m \end{bmatrix}
\]

\[
e = [x \ 0 \ 0] : \text{car going}
\]

\[
e = [0 \ 0 \ 0] : \text{car stopped}
\]
Pedestrian versus smart vehicle problem

Probability \( p \): pedestrian decision and action. 

\( p = 1 \) means pedestrian walk ahead

\[ \gamma = \begin{bmatrix} \bar{p} & 0 & \bar{p} \\ 0 & 0 & 0 \\ \bar{p} & 0 & \bar{p} \end{bmatrix} \]

\[ \gamma \cdot e \implies \]

Each time step we solve:

\[ \gamma_u^v e_v = g_{uw} J^w \]
Payoff matrix of the pedestrian versus smart vehicle 2 actors game

<table>
<thead>
<tr>
<th>Smart Vehicle V</th>
<th>Go ahead A</th>
<th>Stop R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedestrian π</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Go ahead A'</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Stop R'</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\tilde{g} = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}
\]

\[
[\tilde{e}_1] = \tilde{g} \begin{bmatrix} P(V = A|p) \\ P(V = R|p) \end{bmatrix}
\]
Final system to solve and p parameter exchange

\[
\begin{align*}
\gamma(p)e &= gJ \\
\tilde{e} &= \tilde{g}P(p)
\end{align*}
\]

- Car going
- Influence hope of earnings
- Change car behavior
- Change p value

\[ p = \frac{1}{1 + e^{-\alpha[\tilde{e}_1 - \tilde{s}]}} \]

xTAN models a coupled human – machine system
And now, what becomes the system under electromagnetic disturbances?

\[
\begin{align*}
\gamma(p). e &= g.J \\
\tilde{e} &= \tilde{g}. P(J|p) \\
p_+ &= f(\tilde{e})
\end{align*}
\]

As the pedestrian is coupled with the smart vehicle, both are disturbed by electromagnetic interactions. Even if the pedestrian trusted in the behavior of the car, this confidence is destroyed by its unforeseen actions.
In the example given, the xTAN method has shown the capacity to couple:
1. Material modelling
2. Psychological modelling

Future applications have to be done to improve the method in large cyber-physical systems cases.

At IRSEEM, first cases imagined:
✓ Mobility: interaction between a smart vehicle and persons
✓ Smart grid: energy usages
✓ Silver economy & firm 4.0: cobotics

Thank you!